

Closing Wed: HW9A, 9B (9.3, 9.4)  
Final: March 10<sup>th</sup>, 1:30-4:20 in KANE 210  
Comprehensive (8-10 pages).  
There will be two pages on ch 9.

Ch. 9: Be able to

1. Solve separable diff. eq.
2. Use initial conditions & constants.
3. Set up and do ALL the applied problems from homework.

*Worried about applied problems?*

Pay attention today and Monday in lectures. Know the homework well. And go thru my review sheets and look at old finals.

### *Newton's Cooling Law Experiment*

Hot water is in the cup. We will try to predict the temp. at the end of class.

1<sup>st</sup> measurement:

Time =

Temp =

2<sup>nd</sup> measurement:

Time =

Temp =

## 9.4 Differential Equations Applications

### 1. *Law of Natural Growth/Decay:*

Assumption: *“The rate of growth/decay is proportional to the function value.”*

$$\frac{dP}{dt} = kP \text{ with } P(0) = P_0$$

*Example:*

A pop. has 500 bacteria at  $t=0$ .

After 3 hrs there are 8000 bacteria.

Assume the pop. grows at a rate proportional to its size. Find  $B(t)$ .

*Example:*

The *half-life* of cesium-137 is 30 years.

Suppose we start with a 100-mg sample.

Find  $m(t)$ .

*Example:*

Bob deposits \$2000 into a savings account. The money grows at a rate proportional to its size (*i.e.* compound interest). The balance in 4 years is \$2100.

Find the formula  $A(t)$  for the amount in his account in  $t$  years.

## **2. Newton's Law of Cooling:**

Assumption: *“The rate of temperature change is proportional to the difference between the temperature of the object and its surroundings.”*

### **3. Mixing Problems:**

Assume a vat of water has a *contaminant* entering at some rate and exiting at some rate, then

*“The rate of change of the contaminant is equal to the rate at which the contaminant is coming IN minus the rate at which it is going OUT.”*

*Example:*

A 10 Liter vat contains 2kg of salt.

Salt water (brine) containing 3kg of salt per liter enters the vat at 5 L/min.

The vat is well mixed.

The mixture leaves the vat at 5L/min.

Let  $y(t)$  = kilograms of salt in the vat at time  $t$ .

(a) Find  $y(t)$ .

(b) Find the limit of  $y(t)$  as  $n \rightarrow \infty$ .

## Key Notes:

$V$  = volume of the vat (liters)

$t$  = time (min)

$y(t)$  = amount in vat (kg)

$\frac{dy}{dt}$  = rate (kg/min)

$\frac{dy}{dt} = \text{Rate In} - \text{Rate out}$

$$= \left( ? \frac{kg}{L} \right) \left( ? \frac{L}{min} \right) - \left( \frac{y}{V} \frac{kg}{L} \right) \left( ? \frac{L}{min} \right)$$

$y(0) = ? \text{ kg}$

*Example:*

A 100 Liter vat contains 5kg of salt.

Two pipes (A & B) pump in salt water.

*Pipe A:* 3L/min with 4kg of salt/L.

*Pipe B:* 5L/min with 2kg of salt/L.

The mixture leaves the vat at 8L/min.

Let  $y(t)$  = the amount of salt in the vat at time  $t$ .

(c) Find  $y(t)$ .

(d) Find the limit of  $y(t)$  as  $n \rightarrow \infty$ .



#### 4. Air Resistance:

A skydiver steps out of a plane that is 4,000 meters high with an initial downward velocity of 0 m/s. The skydiver has a mass of 60 kg.

(Treat downward as positive).

Let  $y(t)$  = "height at time  $t$ "

Let  $v(t) = y'(t)$  = "velocity at time  $t$ "

Let  $a(t) = v'(t) = y''(t)$  = "accel. at time  $t$ "

*Newton's 2<sup>nd</sup> Law says:*

(mass)(acceleration) = Force

$$m \frac{d^2 y}{dt^2} = \text{sum of forces on the object}$$

The force due to gravity has constant magnitude (and it is acting downward):

$$F_g = mg = 60 \cdot 9.8 = 588 \text{ N}$$

*One model for air resistance*

The force due to air resistance (*drag force*) is proportional to velocity and in the opposite direction of velocity. So

$$F_d = -k v \text{ Newtons}$$

Assume for this problem  $k = 12$ .

## *The Logistics Equation*

Consider a population scenario where there is a limit to the amount of growth (spread of a rumor, for example).

Let  $P(t)$  = population size at time  $t$ .

$M$  = maximum population size.  
(capacity)

We want a model that

...is like natural growth when  $P(t)$  is significantly smaller than  $M$ ;

...levels off (with a slope approaching zero), then the population approaches  $M$ .

One such model is the so-called logistics equation

$$\frac{dP}{dt} = kP \left( 1 - \frac{P}{M} \right) \text{ with } P(0) = P_0$$

Random old final questions:

**Spring 2011 Final:**

Brief summary of what it says:

$v(t)$  = velocity of an object

$$F = mg - kv$$

Recall:

$$F = ma = m \frac{dv}{dt}$$

You are given  $m$ ,  $g$ , and  $k$  and asked for solve for  $v(t)$ .

**Spring 2014:**

A lake has a volume of 1000 cubic meters and contains pure water. A farm near the lake begins using a pesticide. Runoff from the farmland into the lake is 10 cubic meters of water per day, with a concentration of 50 grams of pesticide per cubic meter of water. The lake drains to the ocean at a rate of 10 cubic meters per day.

## Winter 2011

Your friend wins the lottery, and gives you  $P_0$  dollars to help you pay your college expenses. The money is invested in a savings account that earns 10% annual interest, compounded continuously, and you withdraw the money continuously (a pretty good approximation to reality if you make regular frequent withdrawals) at a rate of \$3600 per year.

## Fall 2009

The swine flu epidemic has been modeled by the Gompertz function, which is a solution of

$$\frac{dy}{dt} = 1.2 y (K - \ln(y)),$$

where  $y(t)$  is the number of individuals (in thousands) in a large city that have been infected by time  $t$ , and  $K$  is a constant.

Time  $t$  is measured in months, with  $t = 0$  on July 9, 2009.

On July 9, 2009, 75 thousand individuals had been infected.

One month later, 190 thousand individuals had been infected.

1. 500 bacteria are in a dish at  $t=0$ hr.  
8000 bacteria are in the dish at  $t=3$ hr.  
Assume the population grows at a  
rate proportional to its size.  
Find the function,  $B(t)$ , for the  
bacteria population with respect to  
time.

2. The *half-life* of cesium-137 is 30 years. Suppose we start with a 100-mg sample. The mass decays at a rate proportional to its size.

Find the function,  $m(t)$ , for the mass with respect to time.



3. You invest \$10,000 into a savings account and never make any deposits or withdrawals. The balance grows at a rate proportional to its size (i.e. *interest* is a percentage of the balance at any time). In 3 years, you notice your balance is \$10,400.

Find the function,  $A(t)$ , for the amount of money in the account with respect to time.